Exam Automated Reasoning

Thursday, 4 November 2004, 14 - 17 h.

NB. The exam will be corrected and graded by humans, not by a computer. Therefore, you need not to bother too much about the syntactical peculiarities of PVS and Promela.

1. Let $f : N \to N$ be a function that is ascending, i.e. $\forall m, n \in N(m \leq n \to f(m) \leq f(n))$. Then the following property holds:

$$\forall n \in N(f(0) + n \leq f(n) \ \vee \ \exists m \in N(m < n \wedge f(m) = f(m+1)))$$

Formulate a PVS-theory and a lemma that expresses this property. Then sketch how this lemma can be proved with PVS. Hint: use induction over n.

2. Consider the following situation:

> Four persons (Alan, Bess, Chris and Dave) are in a cave, with one oil lamp. They must cross a narrow passage where only one or two people can go at the same time. It is not safe to be in the passage without light. They can pass the passage with different speeds: Alan in 5 minutes, Bess in 10, Chris in 20 and Dave in 25. When two people pass the passage together, the time needed is the maximum of their times. The oil in the lamp will provide light for 60 minutes.
> Is it possible for them to pass the passage safely? And if so, how can they do it?

Write a Promela program that models this situation, in such a way that the questions can be answered by using SPIN. Your program should be generic, so that it may be generalized to situations with another number of persons, with different passage times. Indicate how to use SPIN to answer the questions. (You are not asked to answer the questions.)

Hints: (1) strive for an assert violation; (2) the following definition may be useful to choose a natural number i with $0 \leq i < n$:

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#define choose(i) atomic{i=0 ; do :: i < n-1 -> i++ :: break od}
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